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Solución al problema de localización de las fuentes cerebrales de las señales del electroencefalograma

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Resumen

En este trabajo se propone una nueva metodología para resolver el problema de localización de fuentes de las señales EEG considerando su comportamiento dinámico. Los métodos usuales ofrecen soluciones estáticas, sin embargo las señales y las fuentes del EEG son dinámicas. Se propone el uno de Filtros de Kalman y Filtros de Partículas para seguir en el tiempo el estado de las fuentes que producen el EEG. Estas dos soluciones proveen una metodología recursiva para predecir y estimar el estado de un sistema dinámico que no se puede observar y solamente se pueden obtener mediciones con ruido relacionadas con el estado. El problema de EEG tiene estas características.

Se uso el modelo de inferencia Bayesiana para modelar el problema de localización de las fuentes del EEG. Posteriormente se derivo la solución del filtro de Bayes la cual provee una forma recursiva de estimar y actualizar la densidad que modela las fuentes desconocidas. En un primer análisis se consideró que el modelo es lineal, que las fuentes y las mediciones se describen mediante distribuciones Gaussianos y que el ruido también es Gaussiano. Para este caso el Filtro de Kalman provee a solución óptima al problema. En un segundo análisis, se consideró que el modelo es no lineal y que los las fuentes y las mediciones son se describen mediante distribuciones no Gaussianas. Para este caso la solución del filtro de Bayes requiere el uso de técnicas de aproximación numérica como los métodos secuenciales de Monte Carlo donde el Filtro de Partículas se puede usar para aproximar de forma recursiva la densidad que describe las fuentes.

Se realizo una evaluación del los métodos dinámicos usando señales EEG simuladas y generadas por fuentes conocidas dentro de la cabeza. Se uso un modelo esférico de tres capas para representar la cabeza.

Además, se establecieron tres condiciones neurofisiológicas para simular las fuentes, primero una fuente superficial, segundo una fuente profunda y por ultimo dos fuentes no correlacionadas. Se propusieron métricas para evaluar el desempeño de las soluciones dinámicas propuestas. Se encontró que las soluciones dinámicas ofrecen un buen desempeño para estimar las fuentes y que mejor rendimiento es mejor que la solución lineal estática del problema.

Dynamic solution to the EEG source localization problem using Kalman Filters and Particle Filters

Abstract

In this work we propose a new methodology for solving the so-called EEG source localization problem considering its dynamic behaviour. Typical methods to solve this problem provide instantaneous or static solutions whereas the EEG signals and sources are dynamic. We propose the use of Kalman filters and Particle filters to dynamically tracking the neural sources that produce the EEG voltages. Both Kalman filter and Particle filter provide a recursive methodology for predicting and estimating the state of a dynamical system that cannot be observed, and only noisy measurements related to the state can be detected. The EEG source localization problem fulfills these characteristics. We use the Bayesian inference to model the EEG inverse problem, where a posterior probabilistic density of possible solutions is obtained and updated at each time given new measurements. In this framework we derive the Bayes filter solution, which provides a recursive way to compute the posterior, besides this posterior can be updated given new measurements. On a first step, we consider that the model of the EEG inverse problem is linear, that the probability distributions of the source and measurements space are Gaussian and that the noise is also Gaussian, for this case the Kalman filter is the best tool to solve the Bayes filter. On a second step we consider that the model EEG inverse problem is non-linear, and that the densities of the source and measurements space can be non-Gaussian, for this case the solution of the Bayes filter requires the use of numerical approximation techniques based on sequential Monte Carlo methods where particle filters can be used to carry out the on-line approximation of the posterior density given the non-stationary sequential measurements.

We evaluate the performance of the proposed dynamic methods by using EEG simulated data generated by known sources inside the head. We used a three-shell spherical concentric head model to represent the head. Three neurophysiological conditions were established to simulate the sources, a single superficial source, a single deep source and two uncorrelated sources. We propose two types of metrics to asses the performance of our dynamic EEG inverse solutions. On the basis of this evaluation we found that our proposed dynamic solution correctly estimates the unknown sources and that they have best performance than the linear static inverse solution.

Contents

Chapter 1

Introduction

Brain imaging techniques such as electroencephalography (EEG), magnetoencephalography (MEG), functional magnetic resonance imaging (fMRI) or positron emission tomography (PET) are powerful technologies to study the brain functionality. They can be used *i*) to diagnose, characterize and treat illnesses and neurological disorders such as epilepsy, schizophrenia, depression, parkinson, alzheimer, tumors, etc., *ii*) to study cognitive processes to understand the brain functions [2], and *iii*) to give people with motor disabilities a new communication channel without using the peripheral nerves and muscles or Brain-Computer interfaces [18].

Although fMRI and PET have very good spatial resolution (the characteristic that allows to see the level of detail in space), as high as 1 to $3mm$, they have very low temporal resolution (the characteristic that allows to see the level of detail in time), as high as $1s$ to several minutes. Also, these technologies are very expensive and they are non-portable. In contrast, technologies such as EEG and MEG have poor spatial resolution, in the order of centimeters, but they have very good temporal resolution, in the order of few milliseconds, besides, they are non-invasive, and EEG in particular is very cheap and portable. Hence, EEG is the most used technique for brain-computer interfaces.

The main source of the brain electrical potentials recorded with electrodes placed in the scalp, or EEG signals, is the synchronous activity of thousands of cortical neurons. A single scalp electrode measures the mix of the temporal activity of groups of neurons at very different and separate brain regions, for this reason, clinical and functional interpretation of EEG signals implies the speculation of the possible active areas within the brain that generate those signals. EEG source localization (also refereed as EEG inverse problem) attempts to find from EEG signals which regions of the brain are active, or to find where the sources are given voltages or measurements at the scalp. This is a transcendental problem in neuroscience which have not been solved yet. The solution to this problem is very important because it could give a non-invasive access to brain function for diagnosis, treatment, understanding [11], [10], and even for brain-computer interfaces applications.

There are two general approaches to solve the EEG inverse problem the dipolar methods and the distributed methods.

Although the large majority of methods fall in these categories, all of them share the property of providing instantaneous or static solutions. These methods do not take into account the time evolution or dynamics of the neural sources. They only use the information from one instantaneous measurement to solve the problem, whereas the EEG sources and the measurements clearly have a time-dynamic structure.

To account with this limitation, in this work we propose a new methodology for solving the so-called inverse problem in EEG considering its dynamic behavior. We propose the use of Kalman filters and Particle filters to dynamically tracking the neural sources that produce the EEG voltages. These two techniques have been widely used to solve dynamic problems in many fields such as robotics or image processing. Both Kalman filter and Particle filter provide a recursive methodology for predicting and estimating the state of a dynamical system that cannot be observed, and only noisy measurements related to the state can be detected. The EEG inverse problem fulfill these characteristics, it is inherently a dynamic system where the sources space cannot be observed directly, and it is possible to obtain noise measurements that are related with the sources. Kalman filter is best dynamic solution for tracking when the model is linear and the variables have a Gaussian distribution. When the model is non-linear or the variables are non-Gaussian analytical solutions are not available and Particle filter can be used to solve tracking problems.

This document is distributed as follows: chapter 2 describes an introduction to the problem and the most widely used static approaches (they do not address the time varying structure of the problem). Chapter 3 proposes the bayesian formulation and the particular solutions named the Kalman filter and Particle filter. Chapter 4 presents a comparison of one commonly used static technique and the two proposed dynamic techniques within three typical setups, with an special emphasis in on the Kalman filter. The results confirm that the two dynamic solutions perform the best in the majority of the cases, specially the Kalman filter in basic situations and more constrained assumptions (linearity and Gaussianity)

Chapter 2

Review of literature

2.1. Physiological basis of EEG signals

EEG signals are a direct consequence of internal electrical currents associated with the synchronized firing of neurons. Nonetheless, one neuron by itself generates a small amount of electrical activity that can not be picked up by electrodes in the scalp. Only when a large group of neurons is synchronous active, the electrical current activity is large enough to be measure as electric potentials by the electrodes in the scalp.

The electrical current flow is the result of negative and positive ionic flow through cells membrane moving from the extracellular to the intracellular space and viceversa [4]. In this way, the total current flow in a particular region of the brain j is the sum of the neuronal activation or primary currents j^P, and secondary currents j^S that result from the electric field induced in the conductive medium due to the primary currents, see equation 2.1. Both primary currents (also known as active or impressed currents), and secondary currents (also known as passive, volume or return currents) contribute to the electric potentials recorded with electrodes in the scalp.

$$j = j^P + j^S \tag{2.1}$$

Since j^S are generated by electric fields induced in the medium, they can be expressed as ohmic currents that depends on the electric field and the conductivity of the medium, so that the total current that generate electric potential is given by 2.2, where $\sigma(r)$ is the conductivity profile of the head tissues.

$$j = j^P + \sigma(r)E \tag{2.2}$$

2.1.1. Physics of the brain electrical activity

Like any other phenomenon involving static or dynamic charge flow, Maxwell's equations and the continuity equation describe the behavior and relation of currents and fields inside the brain.

These formulation allow us to fully model and describe the currents, electric potentials and electromagnetic fields within the head produced by the neural firing, nonetheless simplifications can be done. On the one hand, the neuronal activation frequencies are below $100Hz$, on the other hand, the wavelengths associated with these frequencies are much greater than the dimension of the head. In this way, the temporal interdependency between electric and magnetic fields can be ignored (the time derivatives of the EEG signals are sufficiently small to be neglected). From this analysis, we can derive the quasi-static version of Maxwell's equations, see equations 2.3, 2.4, 2.5, and 2.6, and the continuity equation 2.7.

$$\nabla \cdot E = \rho/\varepsilon_0 \tag{2.3}$$

$$\nabla \times E = 0 \tag{2.4}$$

$$\nabla \cdot B = 0 \tag{2.5}$$

$$\nabla \times B = \mu_0 j \tag{2.6}$$

$$\nabla \cdot j = 0 \tag{2.7}$$

Where E is the electric field, B is the magnetic field, ρ is the charge density, ε_0 is the permitivity of the free space and μ_0 is the permeability of the free space and j is the total current density.

Now, combining the equation 2.2 that describes the total current capable of generate electric potentials at the scalp, with the equation 2.7 that states that the divergence of the total current is zero (which means that the volume conductor or head is electrically neutral), we obtain:

$$\nabla \cdot \sigma(r)E = -\nabla \cdot j^P \tag{2.8}$$

Likewise, equation 2.4 states that electric field rotational is zero, $\nabla \times E = 0$, so that the electric field is the gradient of a electric potential $E = -\nabla\Phi$.

Substituting this equation in equation 2.8 we obtain the Poisson's equation for the electric potential 2.9.

$$\nabla^2 \cdot \Phi = -\nabla \cdot j^P / \sigma(r) \tag{2.9}$$

This equation fully describe the electric potential measurements at the scalp due to the current generators inside the head taking into account the electric and geometric characteristics of the volume conductor or head tissues. In Poisson's equation, a current source j^P represent the current density generated by a large number of synchronous active neurons, which for simplicity can be approximated to a current dipole (unidirectional current element pumped from a sink to a source) as is shown in equation 2.10.

$$j^P(r) = J\delta(r - r_J) \tag{2.10}$$

Where J represents the total current density enclosed in a specific brain region $J = \int j^P(r)dr$, or the dipole moment (strength and orientation), and r_J represents the source location. In this way, substituting equation 2.10 in Poisson's equation 2.9, yields the expression that relates the EEG sources or current dipoles with electric potentials at the scalp:

$$\nabla^2 \cdot \Phi = -\nabla \cdot J\delta(r - r_J) / \sigma(r) \tag{2.11}$$

Brain activity does not actually consist of discrete sets of physical sources, but the dipole is a convenient representation for activation of a large number of neural cells. It is quite complicated to solve analytically the equation 2.11 in a simplistic geometry, for example in spherical models with homogeneous conductivity profiles. In more complicated geometries as the human head where the tissues have inhomogeneous and anisotropic conductivity profiles this equation has not closed solution, hence, numerical solution are the only way to solve the Poisson's equation.

2.2. Head modelling

Poisson's equation 2.11 expresses the relationship between the current dipoles or sources and the scalp potentials or measurements taking into account the electric and geometric characteristics of the volume conductor.

Head tissues as cerebro-spinal fluid (CSF), gray matter, skull, scalp, etc., form together the volume conductor where the currents, electric potentials and

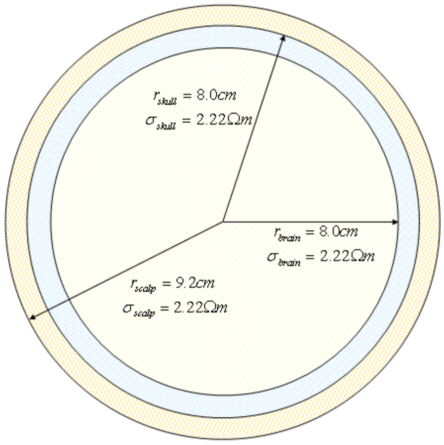

Figure 2.1: Three-shell concentric spherical head model which is composed by three tissues with homogeneous conductivity

electromagnetic fields are produced by the neural activation, so that it is necessary to have a head model that describes the geometry and the conductivity profile for the tissues, and to locate the sources and measurements points.

There are two common approaches: the spherical model and the realistic head model. In the spherical model, the head is divided in three-shell concentric spheres with different conductivities (Figure 2.1). The inner sphere represents the brain, the intermediate sphere represents the skull and the outermost sphere represents the scalp, each one of them with homogeneous but different conductivities. The volume conductor is determined using spherical or realistic models and assigned conductivity values of functions to each type of tissue in the model. These conductivities profiles are typically standard values that have been measured in vitro or in vivo experiments. A good estimate of head conductivity and shape is important because they have a large impact on the model accuracy. This simple model does not consider the anisotropy and inhomogeneity of the tissues. Although this model is a gross approximation, it is quite used because it is easy to implement and it can gives results that correspond reasonably well to the reality.

In realistic head models, the conductivity profile of the head is typically determined by segmenting an anatomical MR image into its various components, e.g., skin, skull, CSF, and brain tissue and assigned to each component a conductivity value o function. Recent studies use up to 10 different tissue

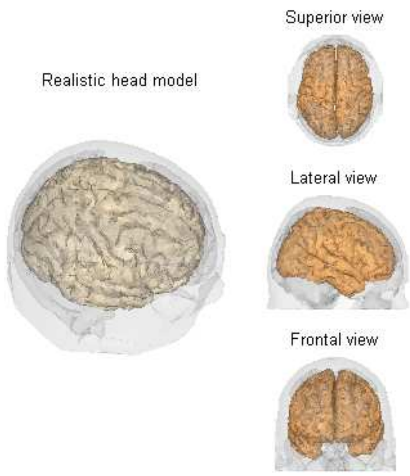

Figure 2.2: Realistic head model formed by three tissues, brain, skull and scalp

types, adding soft bone, fat, eye, cerebellum, muscle, and a separation of white and gray matter. Figure 2.2 shows a realistic head model.

This head model requires the use of sophisticated numerical methods as BEM or FEM, which are based on complex algorithms with high computational load. The boundary element method (BEM) allows realistic modelling of scalp and skull and brain, but not other brain tissues. This method assume that the conductivity profiles are homogeneous for each tissue. The finite element method (FEM) also allows realistic modelling of many tissues including, white matter, gray matter, etc. FEM is based on dividing each tissue in small volumes so that it allows to define anisotropic conductivity profiles. BEM and FEM models assume that the subjects MRI data is available, but this is not always the case, so head atlas such as Montreal Institute Nueoroimaging MNI-158 can be used [7] and [6]. Nevertheless this approach is still an approximation to the real head model for a subject.

2.3. Forward and inverse problem in EEG

In EEG signals and sources analysis we deal with two different but complementary problems, *i*) computing the voltages at different points in the scalp by given current sources inside the head or *forward problem*, and *ii*) computing the current sources inside the head given the voltages in the scalp or *inverse* *problem.*

The goal in forward problem solutions is to calculate the electric potentials in electrodes placed at the scalp given the current density distribution in the brain. In this problem we assume that the sources are completely known and the voltages are the unknown variables to compute. This is a well-posed problem with a unique solution that only depends on the data, and it is always possible to find that solution. However, this problem does not have clinical applications since in practice we do not know a priori the current sources. More formally, the forward problem in EEG states that:

> Given the position, orientation, and the strength of the current sources, as well as the geometry and the electrical conductivity of the head tissues, the forward problem is used to calculate the distribution of the electric potential on the surface of the head or scalp.

On the other hand, the fundamental problem in electroencephalography known as EEG inverse problem, looks for estimate the current sources inside the head given the electric potentials registered in the scalp. In this case, we now a priori the voltage measurements, but the sources are the unknown variables to calculate. This is an ill-posed problem, since given a finite number of scalp potentials, it is possible for an infinite number of current sources configurations to account for those potentials. This means that the inverse problem does not have unique solution. Only an infinite number of measurements sites on the scalp would enable a unique determination of the sources. Formally, the inverse problem in EEG states that:

> Given a discrete set of electric potentials and their locations over the head surface, as well as the geometry and the electrical conductivity of the head, the inverse problem looks for estimates the position, orientation, and the strength of the current sources that generate those voltages.

Figure 2.3 shows graphically both forward and inverse problem in EEG. Although the EEG inverse problem does not have a unique solution, solutions can be approximated by using the forward problem and by imposing physiological and mathematical constrains, for instance, eliminating solutions that fall where is is impossible to find current sources, e.g. outside the head or even in the skull or scalp tissues.

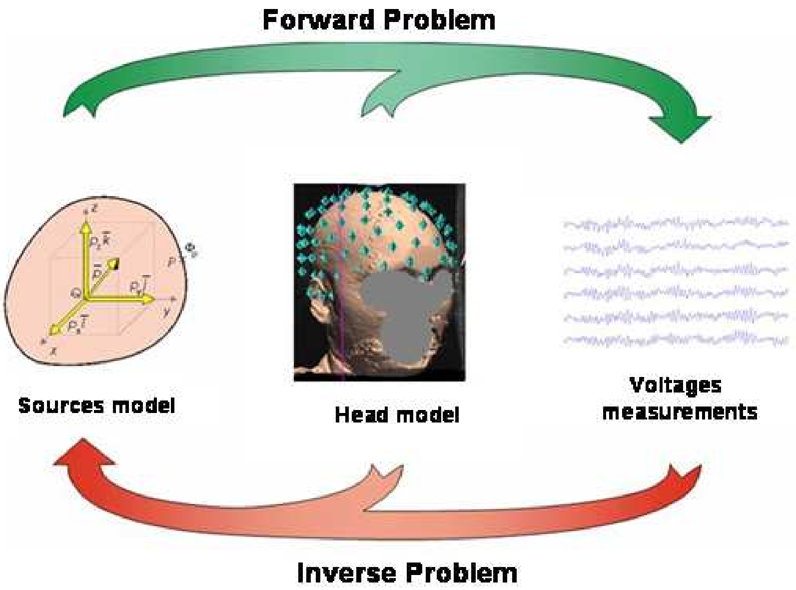

Figure 2.3: Forward and inverse problem in EEG

2.3.1. Modelling and solving the forward problem

If we know the brain activated areas, the electrode locations and the head model (including the geometry and the electric characteristics for each tissue), the forward problem is the solution to the Poisson's equation 2.11. For this equation, we might use the Green's theorem to derive Φ from J. The solution to the Poisson's equation for the simple case of a unique current dipole in an infinite homogeneous isotropic volume conductor is given by equation 2.12, which give us the electric potential Φ at r_s, due to a dipole located at r_J with moment J where σ is the conductivity of the medium.

$$\Phi(r_s, r_J) = \frac{1}{4\pi\sigma} \frac{r_s - r_J}{\|r_s - r_J\|^3} \cdot J \tag{2.12}$$

According to this representation the voltage Φ can be obtained as the inner product between the 1×3 vector $\frac{1}{4\pi\sigma} \frac{r_s - r_J}{\|r_s - r_J\|^3}$ and the 3×1 vector J. First vector, known as lead field vector, gather the sensor position, dipole position, and the medium conductivity, while second vector is the source moment.

However, the most used head model is the three-shell concentric spheres as showed is figure 2.1. For this model, equation 2.13 is the solution of Poisson's equation. Again the voltage can be expressed as the inner product between a 1×3 lead field vector and the 3×1 moment vector.

$$\Phi = \frac{1}{4\pi\sigma_N \|r\|^2} \sum_{n=1}^{\infty} \frac{2n+1}{n} (\frac{r_J}{r})^{n-1} [f_n n \cos\alpha P_n \cos\gamma + g_n \cos\beta \sin\alpha P_n^1 \cos\gamma] \cdot J \tag{2.13}$$

Where P_n and P_n^1 are the Legendre and associated Legendre polynomials, and f_n and g_n are the expressions of the sphere radial and tangential conductivities.

Algebraic formulation of the forward problem

In equation 2.13 the voltage was considered as the inner product between two vectors. Let be $K = k(r_s, r_J, \sigma(r))$ this the lead field, which is a function of the sensor position, dipole position, and the piecewise conductivity. Let be the $\Phi = k(r_s, r_J, \sigma(r))J$ the vector potentials, which is a function of the previous field vector and the source moment J. In matricial representation the forward problem for the case of a single sensor and a single dipole source in a three-shell spherical model is given by 2.14.

$$\Phi_{[1\times1]} = K_{[1\times3]} J_{[3\times1]} \tag{2.14}$$

Consider now the situation where there are various sources activated at the same time. For this case superposition principle can be applied to compute the voltage Φ due to M activated sources, so that the total voltage is the sum of the the individual voltage contribution for each source, $\Phi = \sum_{j=1}^{M} k(r_s, r_{Jj}, \sigma(r))J_j$. Equation 2.15 expresses the forward problem in matricial representation for the case of a single sensor and M dipole sources in a three-shell spherical model.

$$\Phi_{[1\times1]} = K_{[1\times3M]} J_{[3M\times1]} \tag{2.15}$$

However, in practical applications we have M sources and N voltage measurements as is showed in figure 2.4. In this way, the voltage in the $i-th$ sensor is given by $\Phi_i = \sum_{j=1}^{M} k(r_{si}, r_{Jj}, \sigma(r))J_j$. We can write the forward problem as is shown in equation 2.16. This equation called the forward equation, give us the scalp electric potentials as a function of the sources strength and orientation J and the sensors position, sources position, and the geometry with piecewise conductivities profiles or lead field matrix K.

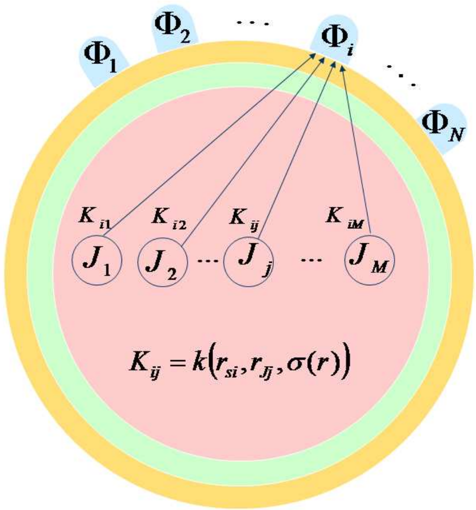

Figure 2.4: Geometric representation of the forward problem. A three-shell spherical head model, M current sources and N voltage measurements

$$\Phi_{[N\times 1]} = K_{[N\times 3M]} J_{[3M\times 1]} \tag{2.16}$$

The forward equation can be extended to include a time component, in this case just Φ and J change their dimensions where each column represent a time moment. Besides, although this analysis was derived for a spherical head model, the same notation remains when using realistic head models only changing the computation of the lead field matrix K.

2.3.2. Approaches to solve the EEG inverse problem

There are two general approaches to solve the EEG inverse problem for the estimation of the EEG sources, the *dipolar methods* also known as parametric, over-determined or scanning methods, and the *distributed methods* also known as non-parametric, imaging, linear and undetermined. Dipolar methods are suggested to be used when a few stationary focal sources are expected to be activated, on the contrary, imaging methods should be used to study more complex activations.

Dipolar methods

These methods are based on iterative solutions of the forward problem, the goal is to optimize the configuration (locations and moments) of a predefined number of dipoles that best explain the voltages measurements. The procedure is to calculate in an starting configuration, the potential produced by the predefined sources (using the forward model) and comparing these results with the potentials measured in the scalp, then to iteratively move until find the best source configuration that explains the measurements. This involves solving the forward problem repetitively and measure the distance between the measured and the calculated potentials. The distance measure most often used to do this is the mean-squared error. Best source configuration is reached when the MSE is minimized.

Notice that to consider just one dipole source is an computational expensive operation, since it is required to span the whole locations in the head. Since one source has six parameter to fix (three for the location and three for the moment), it is required at least six electrodes for each source. For more than one dipole non-linear optimization methods based on directed search algorithms are used. Dipolar methods has many disadvantages, first, they assume that a small number of sources can adequately model the measurements, second, the dipoles locations and moments need to be initialized, third, the optimization can fall in local minimums, and fourth, they are not adequate for real time source localization, however, dipolar methods are preferred in the case of highly focal activation, for example in somatosensory stimulation or in analysis of epileptic brain activity.

Distributed methods

These methods do not make assumptions about the number and location of the sources. They are well suitable to predict large brain activated areas that generate the voltages measurements. The brain space is divided in a $3D$ grid of solution points which number is much greater than the number of measurements ($M >> N$) making the problem undetermined and non-parametric. The goal is to find a unique configuration (strength and orientation) at the static predefined grid points or source location that explain the measurements. This means that each solution point is equivalent to a current source with fixed position that can be activated at any time and independently, only orientation and strengths can vary.

The inverse problem is linear since the only unknown parameters are the dipole moment for each grid point, meaning that a matrix can be constructed that linearly relates the estimated solution with the measured data. The discrete linear solution for the EEG inverse problem can be written as 2.17.

$$\hat{J} = T\Phi \tag{2.17}$$

Where T is a $3M \times N$ matrix that define some generalized inverse of the lead field matrix K explained in the modeling of the forward problem 2.16. Since there are an infinite number of generalized inverse matrices T that produce $\hat{J}'s$ that satisfies the measurements Φ, the EEG inverse problem has infinite solutions. There are various methodologies to derive the operator T for a particular EEG linear inverse solution. In this work we will consider the methodology based on the constrained solution to the forward equation. The most common and used EEG linear inverse solutions are: Minimum norm (MN), Weighted minimum norm (WMN), and Low resolution electromagnetic tomography (LORETA).

The minimum norm solution (MN) is the natural solution for problems without a unique solution and without a priori information. It was initially applied to the EEG inverse problem solution by [16]. This solution states that from all the possible solutions, the most reasonable one is the solution with the lower energy, mathematically, the solution with the minimum L2-norm, as stated in 2.18. The solution to this minimization is the equation 2.17, where T is given by 2.19. The superscript † denotes the Moore-Penrose pseudoinverse.

$$min\|J\|^2, \textit{ under the constrain: } \Phi = KJ \tag{2.18}$$

$$T = K^T[KK^T]^\dagger \tag{2.19}$$

This method gives a unique solution in the sense that only one configuration of the sources can have both characteristics, the lower energy and the generation of the voltage measurements. However, there is not physiological evidence to avoid other solutions, for instance, solutions with the second or the third lower energy could be the one generating the voltages. In this way, MN punish the solutions with a large number of grid points activated at the same time or deeper sources, in favor of the small number and superficial sources. In consequence deeper sources have less weight in the MN solution which is not physiological valid.

The weighted minimum norm solution (WMN)is an improving of the MN which looks for to eliminate the favor of superficial sources given more weight to deeper sources, so that this solution states that all sources produce equal size measurements. In attempt to eliminate the favor of superficial sources, the WMN introduce a weight matrix into the minimization as showed in 2.20. The solution to this minimization is again $\hat{J} = T\Phi$, but now T is given by 2.21.

$$min\|WJ\|^2, \textit{ under the constrain: } \Phi = KJ \tag{2.20}$$

$$T = W^{-1}K^T[KW^{-1}K^T]^\dagger \tag{2.21}$$

Where W is a $3M \times 3M$ diagonal matrix where each diagonal element is chosen to reduce the importance of the grid points that are near to the sensors or head surface. Weight matrix is calculated as $W = \Omega \otimes I_3$, where $\oplus$ denoted the Kronecker product, I_3 is 3×3 identity matrix and Ω is a $M \times M$ diagonal matrix where each diagonal element is given by $\Omega_{ii} = \sqrt{\sum_{j=1}^{N} K_{ji}^T K_{ji}}, \forall i = 1, 2...M$. Nevertheless, the introduction of the weight matrix is based on mathematical operations more than in a physiological basis, besides, solutions obtained with this method can be very blurred.

Low resolution electromagnetic tomography (LORETA) is the most used EEG inverse solution, which is based on the physiological constrain that the activity of sets of neighbor neurons is correlated. This inverse solution is implemented in the free academic software Loreta-Key [12]. In this case the minimization includes a weighted operator B as showed in 2.22, where B is a $3M \times 3M$ matrix that implements the discrete spatial laplacian operator. The solution to this minimization is $\hat{J} = T\Phi$, where T is given by 2.23. Details to derive B can be found in [13].

$$min\|BWJ\|^2, \textit{ under the constrain: } \Phi = KJ \tag{2.22}$$

$$T = (WB^TBW)^{-1}K^T[K(WB^TBW)^{-1}K^T]^\dagger \tag{2.23}$$

This Classical LORETA solution do not take into account anatomical brain distinctions, for example it can give a solution where a brain activated region cover at the same time two adjacent lobes o even both brain hemispheres given over-smoothed solutions that can be physiologically incorrect.

To overcome some of the limitations of this technique, the authors propose sLORETA [14], which is an improvement to address some specific issues related to the non-zero localization.

Chapter 3

Bayesian inference applied to the EEG inverse problem

Dipolar methods are very far from the real neural activation resolution and distributed methods lack of the ability to incorporate physiological and temporal constrains to the inverse solution, so that these methods can produce unreal solutions or solutions with unrealistic smooth. Besides, these methods do not take into account the time evolution or dynamics of the EEG sources, they only use the information from one instantaneous measurement whereas EEG generators and measurements clearly have a time-dynamic structure. To account with these limitation, this thesis proposes the usage of Bayesian inference, where instead of a unique solution, a probabilistic density of possible solutions is obtained and updated at each time given new measurements.

Since the measurements and the sources can be considered as random variables, the inverse problem can be modelled using the Bayes theorem as showed in equation 3.1, with $X = [J, r_J]$ being the source space ($J = [J_x, J_y, J_z]$ represents the moment of the source and $r_J = [r_x, r_y, r_z]$ represents the position of the source), $P(X|\Phi)$ being the posterior probability that models the sources given the voltages measurements, $P(\Phi|X)$ being the likelihood function that contains the forward model and allows to compute the measurements given the sources, and finally $P(X)$ being the prior knowledge about the sources.

$$P(X|\Phi) = \frac{P(\Phi|X)P(X)}{P(\Phi)} \tag{3.1}$$

With the model of the EEG inverse problem in the Bayesian framework the goal is to compute the posterior probability to make inference about the sources, however the sources and the measurements are changing over the time so that a recursive update equation should be obtained to deal with the dynamic behavior of the EEG inverse problem. The Bayes filter provides a recursive solution to compute the posterior distribution and update it given new measurements.

3.1. Bayes filtering in the context of the EEG inverse problem

Since the EEG inverse problem is inherently dynamic, the Bayes filter can be used to estimate recursively the posterior distribution of the sources X_t at time t, from the voltage measurements Φ_t up to time t, or $P(X_t|\Phi_{0\ldots t})$. To derive a recursive update equation, this expression can be transformed by the Bayes theorem to obtain:

$$P(X_t|\Phi_{0\ldots t}) = \frac{P(\Phi_t|X_t, \Phi_{0\ldots t-1})P(X_t|\Phi_{0\ldots t-1})}{P(\Phi_t|\Phi_{0\ldots t-1})} \tag{3.2}$$

We assume that the measurements are Markov, which means that the measurements Φ_t at time t are conditionally independent of past measurements given knowledge of the state X_t, that is $P(\Phi_t|X_t, \Phi_{0\ldots t-1}) = P(\Phi_t|X_t)$, so that the posterior can be written as:

$$P(X_t|\Phi_{0\ldots t}) = \frac{P(\Phi_t|X_t)P(X_t|\Phi_{0\ldots t-1})}{P(\Phi_t|\Phi_{0\ldots t-1})} \tag{3.3}$$

To obtain the recursive form, we now have to integrate out the source X_{t-1} at time $t-1$, which yields:

$$P(X_t|\Phi_{0\ldots t}) = \frac{P(\Phi_t|X_t)}{P(\Phi_t|\Phi_{0\ldots t-1})} \int P(X_t|X_{t-1}, \Phi_{0\ldots t-1})P(X_{t-1}|\Phi_{0\ldots t-1})dX_{t-1} \tag{3.4}$$

The Markov assumption also implies that if the state X_{t-1} is known, the state X_t is conditionally independent of past measurements, that is, $P(X_t|X_{t-1}, \Phi_{0\ldots t-1}) = P(X_t|X_{t-1})$ and $P(X_t|\Phi_{0\ldots t}) = P(X_t|\Phi_t)$. Finally, equation 3.5 gives the Bayes filter for the dynamic EEG inverse problem.

$$P(X_t|\Phi_t) = \eta P(\Phi_t|X_t) \int P(X_t|X_{t-1})P(X_{t-1}|\Phi_{t-1})dX_{t-1} \tag{3.5}$$

Where $\eta = \frac{1}{P(\Phi_t|\Phi_{0\ldots t-1})}$ is a normalizing constant. This equation tell us that the probability of the sources distribution X_t given the measurements Φ_t

at time t, depends on the likelihood function $P(\Phi_t|X_t)$, the dynamic model $P(X_t|X_{t-1})$ and the previous state $P(X_{t-1}|\Phi_{t-1})$. To implement the Bayes filtering we need to know three distributions, the initial posterior $P(X_0)$ that characterizes the initial knowledge about the source, the next state probability or dynamic model $P(X_t|X_{t-1})$ that represents the time evolution of the sources which is assumed to be known, and the measurement model $P(\Phi_t|X_t)$ that allows to obtain measurements given a known source space.

The recursive solution provided by the Bayes filter allows to compute the posterior distribution of a model described by these two equation:

$$X_t = G(X_{t-1}) + w_t \tag{3.6}$$

$$\Phi_t = F(X_t) + v_t \tag{3.7}$$

Where equation 3.6 is known as the process or state model and equation 3.7 is referred as the measurement model. Furthermore, the variables $\{X_t\}_{t=0,1..}$ and $\{\Phi_t\}_{t=0,1..}$ are stochastic variables (they can have any type of probabilistic distribution) that defines the unknown source space and the known measurement space, respectively. Likewise, $G(\cdot)$ defines the relation between the actual source space and the next source state and $F(\cdot)$ defines the relation between the actual source space and the actual measurement. These functions can be linear or nonlinear. Finally, the random variables w_t and v_t represent the noise in the process and in the measurements respectively.

In summary, the model that can be estimated by the solution of the Bayes filter have three main properties: first, the model linearity or non-linearity, second, the form of the distribution of the source and measurements space, and third, the type of the distribution of the additive noise.

In the framework of the EEG source localization problem, the model can fall in one of these two categories: on the first hand, if we assume a distributed approach where the location of the sources are known, the source space X reduces to $X = J = [J_x, J_y, J_z]$, and the model becomes linear. On the other hand, we can assume a dipolar approach where it is necessary to estimate the sources location and moment $X = [r_J, J]$, which means that the model is non-linear.

For the first case, when the model is linear, we can assume that the probability distributions of the source and measurements space are Gaussian and the noise is also Gaussian, the Kalman filter is the best tool to solve the Bayes filter since it is the optimal estimator. Nevertheless, for the second case where the model is non-linear, and the densities of the source and measurements space can be non-Gaussian, the solution of the Bayes filter requires the use of numerical approximation techniques based on Monte Carlo methods.

These two dynamic techniques will be now applying in the EEG source localization problem.

3.2. Applying the Kalman filter to solve the EEG inverse problem

If we assume that the EEG inverse problem is a linear (distributed approach) and Gaussianity for the variables, it is possible to solve analytically the Bayes filter equation and to obtain an analytical expression to derive the posterior distribution that models the EEG sources. This solution is known as the Kalman Filter. In the context of EEG source localization, the Kalman filter addresses the problem of trying to estimate the sources or sources state over the discrete-time process that is governed by this linear stochastic equation (see for more details [17]):

$$J_t = AJ_{t-1} + w_t \tag{3.8}$$

Where A is a known matrix representing the transition from the current state of the sources space to the next sources state which is assumed to be known, and w is the noise in the process. This noise can be considered as a Gaussian random variable with zero mean and a covariance matrix Q, so that its probability distribution is $p(w) \sim N(0, Q)$.

Besides the linear model, it is necessary to have a linear measurement equation that relates at each time the voltages and the sources state which is basically the equation of the forward model:

$$\Phi_t = KJ_t + v_t \tag{3.9}$$

Where K is the known as the measurement matrix and v is the noise in the measurements. This noise is considered to be a Gaussian random variable with zero mean and a covariance matrix R so that its probability distribution given by $p(v) \sim N(0, R)$.

In order to estimate the state of the neural sources at time t, the Kalman filter perform recursively two steps, i) a time update step which estimates the next state J_t using the linear model equation, and ii) a measurement update step which adjust the estimated state J_t by using the actual measurement Φ_t. The set of time update and the measurement update equations establish the recursive algorithm or Kalman filter that estimates the sources distributions given noisy voltage measurements.

To implement a dynamic EEG inverse solution based on the Kalman filter we need to define the next properties:

- The dynamic model is the knowledge of how the neural sources evolves over the time. It is impossible to know that. Nonetheless, we can assume random evolving of the sources, which is equivalent to say that the transition matrix A is the identity matrix, so that the initial estimation of the next state is equal to the current state plus a random walk.

- Given the first observation, the estimation of the initial state is given by the classical linear inverse solution.

- The measurement matrix is the lead field matrix which can be easily computed once the head model is defined.

Having defined this three properties, it is possible to execute the Kalman filter algorithm for tracking the neural sources from noisy voltage measurements.

3.3. Applying Particles filter to solve the EEG inverse problem

If we assume that the EEG inverse problem is non-linear (dipolar approach) and possibly non-Gaussianity for the source space and for the measurement space, it is very hard to solve the Bayes filter equation to obtain an analytical expression. In this case sequential Monte Carlo methods offers a numerical approximation for the solution of the Bayes filter where the posterior distribution is represented by a set of samples whose distribution can be used to make inferences about the sources space.

To our knowledge there is not work that solve the EEG inverse problem using MC methods, but there are some approaches to solve the quite similar magnetoencephalography (MEG) inverse problem. See [5] and [3] for more details. Within the various Monte Carlo Methods, the particle filters can be used to carry out the on-line approximation of the posterior density $P(J_t|\Phi_t)$ given the measurements Φ_t. This technique is appropriate to deal with the dynamic non-linear EEG inverse problem because the voltages and the sources are non-stationary, because the measurements arrive sequentially and because the densities of the source space and the measurement space can be assumed to be non-Gaussian (for more details see [1]).

In the context of EEG source localization, the Particle filter addresses the problem of trying to estimate a set of samples (named particles) that represent the posterior distribution given by $P(X_t|\Phi_t)$. The idea is to represent the posterior probability by a set on N samples or particles $\{X^{(i)}, w^{(i)}\}_{i=1,\dots,N}$, where each particle $X^{(i)}$ is a possible source distribution, and each $w^{(i)}$ is a non-negative importance factor that determine the weight of its corresponding particle. In execution, the set of particles $\{X_{t-1}^{(i)}\}_{i=1,\dots,N}$ is distributed according to $P(X_{t-1}|\Phi_{t-1})$, then, by using the dynamic model we obtain a new set of particles $\{X_t^{(i)}\}_{i=1,\dots,N}$ which is distributed according to $P(X_t|\Phi_{t-1})$ where the dynamic model specifies how the sources evolves from $t-1$ to t. Then, the importance weights $w_t^{(i)}$ are computed through the likelihood function, $w_t \propto P(\Phi_t|X_t)$. A resampling step is applied to the weighted sample $\{X_t^{(i)}, w_t^{(i)}\}_{i=1,\dots,N}$. After this, we obtain a new sample set $\{X_t^{(i)}\}_{i=1,\dots,N}$ distributed according to the posterior density $P(X_t|\Phi_t)$. Finally, the sample $\{X_t^{(i)}\}_{i=1,\dots,N}$ can be used to make inference of the sources.

To implement a particle filter in the context of EEG source localization, we need to know i a initial distribution or prior density for the sources $P(X_0)$ which can be uniform distributed within the range of the sources, ii) a dynamic model $P(X_t|X_{t-1})$ that describes the time evolution of the sources and ii) the likelihood density or measurement model $P(\Phi_t|X_t)$ that allows to compute the voltages given a set of sources.

Although the potentiality of the particle filter is for non-linearity and non-Gaussianity, in the framework of this thesis we will simplify the election of these properties to work in the same scenery of the Kalman filter. In the future work we will extend the use of the Particle filter in more complicated setups as the non-linear EEG inverse problem (dipolar approach) or non Gaussian noise. According to these ideas, our first implementation of the Particle filter to solve the dynamic EEG inverse is based on the next properties:

- The initial prior density $P(X_0)$ is uniformly distributed in the sources space.
- The dynamic model $P(X_t|X_{t-1})$ of how the neural sources evolves over the time. Similarly to the case of the Kalman filter we can assume random evolving of the sources which is a zero mean Gaussian function representing a random walk of the sources. In other words, simply making each particle to evolve randomly.

- The likelihood function is a zero-mean Gaussian density.
- We can use the mean values of the posterior, to obtain the solution or the sources space.

With these definitions we can run a Particle filter for the dynamical estimation of the EEG sources.

Chapter 4

Evaluation of the methods

After defining the Kalman filter and the Particle filter in the context of the EEG inverse problem, we evaluate their performance by using EEG simulated data generated by known sources inside the head. Different types of sources were carefully designed to follow a time dynamic evolution since the main characteristics of our proposed methods are their ability for tracking the dynamic of the neural generators. Three neurophysiological conditions were established to simulate the sources. Several metrics were designed to evaluate the quality of the estimated solution obtained by our algorithms. Together with the Kalman filter and Particle filter, the instantaneous inverse solution was also computed to estimate the sources given voltage measurements.

4.1. The physical model

We used a three-shell spherical concentric head model to represent the brain, skull and scalp. The radius and the conductivities chosen for each head tissue are summarized in table 4.1, which correspond to typical normalized values for these tissues that have been used in many studies of EEG signals and sources [8].

Additionally, the measurement space consist of 56 real electrodes positions from the 10/10 international system projected to lying on the outermost sphere surface, hence the distance from the coordinate system origin to each electrode is 1. Figure 4.1 shows a graphs of the electrodes lying over the sphere surface.

The solution space consists of 10 grid points arranged in 2 spherical layers with radius of 0.2 and 0.8, and distributed in the upper half of the innermost sphere that represents the brain volume. Each layer contains 5 grid points. Figure 4.2 depicts the grid points within the sphere representing the head.

Table 4.1: Parameters of the three-shell spherical head model used in the evaluation of the methods

Head Tissue	Radius (cm)	Conductivity ($\frac{1}{\Omega m}$)
Brain	0,82	1,0
Skull	0,88	0,0125
Scalp	1,00	1,0

For this physical model the lead field was computed obtaining a matrix K of 56×30. Since the number of dimensions of the measurement space ($N_{elec} = 56$) is greater that the number of dimensions of the sources space ($3 * N_{grid} = 30$) our model is fully determined. Before going to solve the EEG inverse problem using Kalman Filter and Particles Filter in the approach of linear distributed and undetermined models, we have worked with a linear distributed but determined model. This first step is necessary to asses the proposed dynamic methods and to identify their correct parameters in the context of the EEG source localization problem.

4.2. The simulated data

In order to evaluate our dynamic EEG inverse solutions, we simulated active sources within the brain located at some of the fixed grid points defined in the physical model. The moment components (J_x, J_y, J_z) of the active sources were varying with time according to a time function. For each time instant, the corresponding EEG voltages (Φ_t) were calculated simply by multiplying the lead field matrix (K) with the simulated source space (J_t). Then, the measurements were corrupted with Gaussian noise. Consequently, both the true sources space and the EEG signals are known for each time.

These characteristic were considered when the sources were simulated:

- The intensity of each moment component vary between -60nAm and 60nAm. This produces EEG signals between $\pm 100\mu V$.

- The measurements were corrupted with zero mean Gaussian noise with a standard deviation chosen to obtain a SNR of $30dB$, ($SNR = 20 \lg \frac{\sigma_M}{\sigma_N}$, with σ_M being the standard deviation of the obtained measurements and σ_N being the standard deviation of the noise).

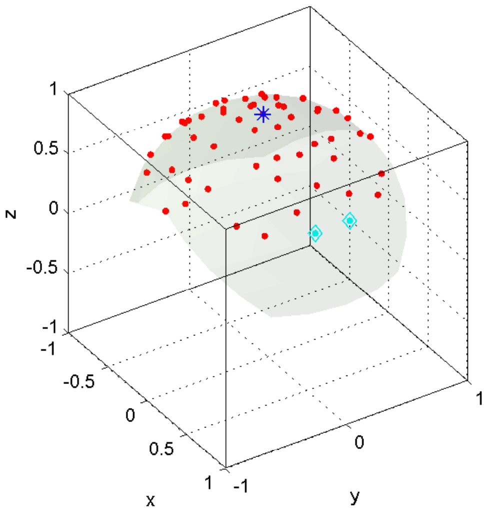

Figure 4.1: Graphical representation of the electrodes position used to compute the measurements for the evaluation of the methods. Electrode marked with "*" represents Cz and electrodes plotted as "$\diamondsuit$" represent $Fp1$ and $Fp2$

- The moment components varies with time as a damped sine function with frequencies in the range to produce EEG signals in the delta and theta bands, (0 to $7Hz$).

- For non active sources the moment component was randomly chosen from a Gaussian distribution with zero mean and standard deviation 1.

- The sample frequency of the EEG signals is $256Hz$ and the time window is 1 second.

We define three types of active sources with neurophysiological meaning, a single superficial source, a single deep source and two uncorrelated sources.

As a superficial source, the grid point located at $[-0,40;0,40;0,57]$ was selected as active source. The deep source was selected to be located at the grid point with coordinates $[0,20;-0,20;0,28]$. Finally, these two grid points were used for the two uncorrelated sources. In the three simulations, the moment components (J_x, J_y, J_z) were varying with time during one second according to different sinusoidal damped time functions. The inactive sources

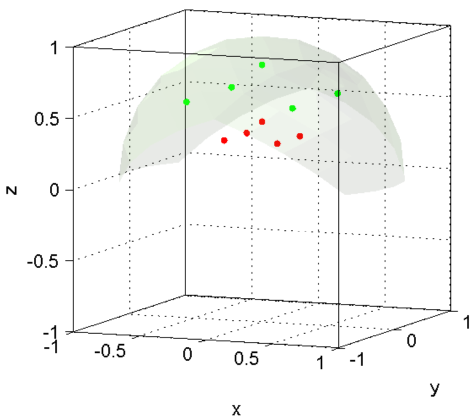

Figure 4.2: Grid points of the solution space consisting of 2 spherical layers of 5 electrodes each.

were corrupted with zero mean and standar deviation of 1 Gaussian noise to simulate background neurophysiological noise.

For these three conditions, the Kalman filter and the Particle filter were used to estimate dynamically the neural sources space. Moreover, we also compute compute the instantaneous inverse solution given by $\hat{J}_{tinverse} = K^{-1}\Phi_t$.

4.3. Metrics

There have been some metrics proposed to evaluate the performance of instantaneous, distributed, discrete, and linear EEG inverse solutions, see [8], [9] and [15]. Based on them we propose two types of metrics to asses the performance of our dynamic EEG inverse solutions, *i*) metrics to evaluate the performance in the estimation of the active sources and *ii*) metrics to evaluate the performance in the estimation of the whole source space. The metrics are computed for each time step and the average is computed to asses the performance during the time window.

4.3.1. Metrics to evaluate the estimation of the active sources

The primary goal of a EEG inverse solution is the localization of the active sources, in this sense we define the *source localization error (SLE)* as the distance between the location of the real active source and the location of the maximum source estimated by the method. Equation 4.1 defines the SLE for a given time step, where r_J is the location of the real active source and $r_{\hat{J}}$ is the location of the estimated source with the maximum moment magnitud. Since our aim is to evaluate the dynamic performance of the EEG inverse solution, we can compute the total or average source localization error as $SLE_{average} = \frac{1}{T}\sum_{t_i} SLE_{t_i}$ where SLE_{t_i} is the SLE at time t_i and T is the total number of time steps.

$$SLE = \|r_J - r_{\hat{J}}\| \tag{4.1}$$

Furthermore, we define the *estimation error (EE)* as the absolute error between the magnitude of the real active source and the magnitude of the estimated source at the same location as showed in equation 4.2, where J_{r_J} is the moment of the real active source located at r_J and $\hat{J}$ is the moment of the estimated source also at the location r_J. Similarly, during a time window the average estimation error is defined as $EE_{average} = \frac{1}{T}\sum_{t_i} EE_{t_i}$, where EE_{t_i} is the EE at time t_i and T is the total number of time steps.

$$EE = \frac{\|J_{r_J}\| - \|\hat{J}_{r_J}\|}{\|J_{r_J}\|} \tag{4.2}$$

4.3.2. Metrics to evaluate the estimation of the sources space

The second goal in an EEG inverse solution is to reconstruct the total sources space as well as possible, in this sense we define the *reconstruction error (RE)* as the absolute error between the total real sources space and the total estimated sources space as given by equation 4.3. The average reconstruction error that gather the total error of the estimated sources space in a time window is given by $RE_{average} = \frac{1}{T}\sum_{t_i} RE_{t_i}$.

$$RE = \frac{1}{N_{grid}} \sum_{i=1}^{N_{grid}} \frac{\| J_i - \hat{J}_i \|}{\| J_i \|} \tag{4.3}$$

Another important metric to evaluate the performance in the estimation of the total sources space is the *data fit error (DFE)*, which is defined as the absolute error between the real voltage measurements and the voltages obtained with the estimated source space as defined by equation 4.4. Similarly, for a time window, the average data fit error is given by $DFE_{average} = \frac{1}{T}\sum_{t_i} DFE_{t_i}$.

$$DFE = \frac{1}{N_{elec}} \sum_{i=1}^{N_{elec}} \frac{\parallel \Phi - K\hat{J} \parallel}{\parallel \Phi \parallel} \tag{4.4}$$

4.4. Results analysis

We report here the results obtained by the three inverse solutions (instantaneous inverse solution, Kalman Filter and Particle filter) in the simulation of the three predefined neurophysiological conditions (a single superficial source, a single deep source and two uncorrelated sources). Table 4.2 shows the values of the average metrics for the three experiments. Figure 4.3 shows the in the left column the magnitude of the real active source and the source magnitude estimated by the three methods. This figure also shows in the right column the estimation error in the total source space.

Table 4.2: Values of the metrics obtained in the simulations

Source type	Average value of	$\hat{J}_{Inverse}$	$\hat{J}_{Kalman}$	$\hat{J}_{Particle}$
Superficial	SLE	62,16	2,46	0.00
	EE	4,91	1,47	7,37
	RE	36,68	21,37	35,26
	DFE	2,21	3,11	10,77
Deep	SLE	23,48	0,24	0,12
	EE	96,68	13,44	29,94
	RE	36,91	20,50	31,90
	DFE	4,32	6,21	11,26
Uncorrelated	SLE1	72,04	21,07	0,00
	EE1	5,24	2,02	9,08
	SL2	21,16	64,38	77,57
	EE2	98.86	14,66	27,30
	RE	33,50	19,92	26,35
	DFE	2,11	3,01	10,72

4.4.1. Superficial source

The first characteristic to evaluate is the performance in the localization of the active sources. Figure 4.3a shows the real source moment magnitude and the results obtained with the three inverse solutions in the case of a single active superficial source. Notice that the three inverse solutions track the real solution. The dynamic Kalman filter solution produces the source magnitude more similar to the real one. The instantaneous inverse solution also tracks correctly the real magnitude, however the estimation is very noisy (this is due to the fact that this solution is more sensitive to the measurement noise). Finally, the dynamic Particle filter solution produces a very good tracking but there is difference between the real values and the estimated values.

The upper part of the table 4.2 shows the average source error localization and the average estimation error given by the three solutions. Since the particle filter is the only methods that has zero $SLE_{average}$, this solution has a very remarkable performance localizing the real active superficial source. On the other hand, the instantaneous inverse estimation has a very high $SLE_{average}$ indicating that most of the times this solution produces the greater source magnitude in other location different to the real one (The performance of this solution is poor in localizing the active source). The $EE_{average}$ is very low for the Kalman filter, hence this solution has the best performance in the estimation of the active source magnitude. Although the $EE_{average}$ for the particle filter is almost 8%, this result just indicates that some times the values of the estimated active source magnitude are not very close with the real values.

The other aspect in the performance of the inverse solutions is the quality in the reconstruction of the total sources space. Since the values for the $RE_{average}$ are around 30%, we can say that the reconstruction of inactive sources (remaind that for each time instant the values of these sources are randomly chosen form a normal distribution with zero mean and a standard deviation of one) is not very precise, however this finding is not critical if the total source space given by the solution produces measurements very similar to the real measurements. In this sense, the values in the $DFE_{average}$ metrics are low, thus, we can say that all the solutions produce estimations on the whole sources space that explain very well the real measurements.

Finally, figure 4.3d shows the estimation error of the whole source space. Sources 1 to 5 correspond to deep inactive generators, sources 6 to 10 correspond to superficial generators. Source 9 is the active generator. The fact that the two dynamic solutions have the lowest estimation errors in comparison with the instantaneous inverse solution suggest that Kalman Filter and Particle filter estimated better the total source space. On the other hand, we can notice that all the solutions have a remarkable estimation of the active source.

4.5. Deep source

Figure 4.3b shows the real and the estimated sources magnitude for the case of a single active deep source. The estimation given by the instantaneous inverse solution is very poor, which is a expected behavior in linear inverse estimators of EEG generators where the mathematical solution punish deep sources. Only the Kalman filter and the particle filter track correctly the real solution, nonetheless it can be notice differences in the magnitude between the real solution and the estimated solution. These differences are due to the fact that the estimation of the initial estate of the filters is precisely the linear inverse solution.

The average source localization and estimation errors given by the three solutions are shown in the middle part of the table 4.2. In this case, the Kalman filter has the best performance in the estimation of the location of the real active deep source (Lower $SLE_{average}$) which means that most of the times, this solution produces the greater source magnitude in the location of the real active source. Particle filter has also a good performance. The instantaneous inverse solution has the bigger $SLE_{average}$ indicating that this solution in the less appropriate to localize deep sources. On the other hand, Kalman filter and Particle filter have both very good performance in the estimation of the magnitude of the deep source ($EE_{average}$ values are around 15%), however the $EE_{average}$ is very high in the instantaneous inverse estimation, which suggest again that this technique is not appropriate for the estimation of deep sources.

To asses the performance in the reconstruction of the total sources space, the results of the average reconstruction error and the average data fit error given by the three algorithms are also showed in the middle part of the table 4.2. Firstly, values for the $RE_{average}$ are very high, so that the sources space estimated by the algorithms are very different in comparison with the real sources space. Two reasons explain this behavior, on the one hand, the estimation of active deep sources is poor since the three algorithms are based on the linear inverse solutions which punish deep sources in favor of superficial sources. On the other hand, the reconstruction of inactive sources is not very precise since their time evolution follow a random normal distribution instead a time function. As in the case of superficial sources this result is not critical if the total source space given by the solution produces measurements very similar to the real ones and the estimation of the active sources is good. Secondly, the values in the $DFE_{average}$ metrics are low indicating that the source space estimated explain correctly the real measurements. Notice that although the linear inverse solution has the worst reconstruction error, the estimated source space produce measurements very similar to the real ones. On the contrary, $DFE_{average}$ values are slightly greater for the Kalman and Particle filter in comparison with the inverse estimator, however, this is not critical since the two dynamic inverse solutions produces sources space more similar to the real one.

Finally, figure 4.3e shows the estimation error of the whole source space where the source 2 is the active dipole. Similarly with the case of superficial sources, in comparison with the instantaneous inverse solution, the two dynamic solutions have the lowest estimation errors in all the sources, thus Kalman Filter and Particle filter estimated much better the total source space. Furthermore, notice how the estimation of the deep active source is very remarkable in the three methods. In contrast, the estimation of inactive deep sources is poor for the instantaneous inverse solution.

4.6. Two uncorrelated sources

Real sources moment magnitude and the results obtained with the three inverse solutions for the case of the two uncorrelated sources (one deep and the other superficial) are shown in figure 4.3c. Results are very similar to the results obtained previously in the single cases. Average reconstruction error and the average data fit error given by the three algorithms are showed in the lower part of the table 4.2. We can see that Kalman filter and Particle filter have the lowest values in most of the metric indicating that these two methods produce sources spaces very similar to the real source space. On the contrary, the performance of the instantaneous inverse solution in reconstructing the whole source space is very poor since all the time the reconstruction error is around 33%. on the other hand, the Kalman Filter and the instantaneous inverse solution estimate sources spaces that generate measurements quite similar to the real measurements. The Particle filter produce slightly greater values of the DFE, but again this is not critical since the source space estimated remains quite similar to the real one.

Finally, figure 4.3f shows the estimation error of each source, where sources 2 and 9 are the active dipoles. It can be seen that Kalman Filter and Particle filter have a very good performance in the estimation of the total source space (Estimation error is always lower than 20% for all the sources). Again, the instantaneous inverse solution has a poor performance in the estimation of inactive deep sources (Estimation error is around 70% for inactive deep sources).

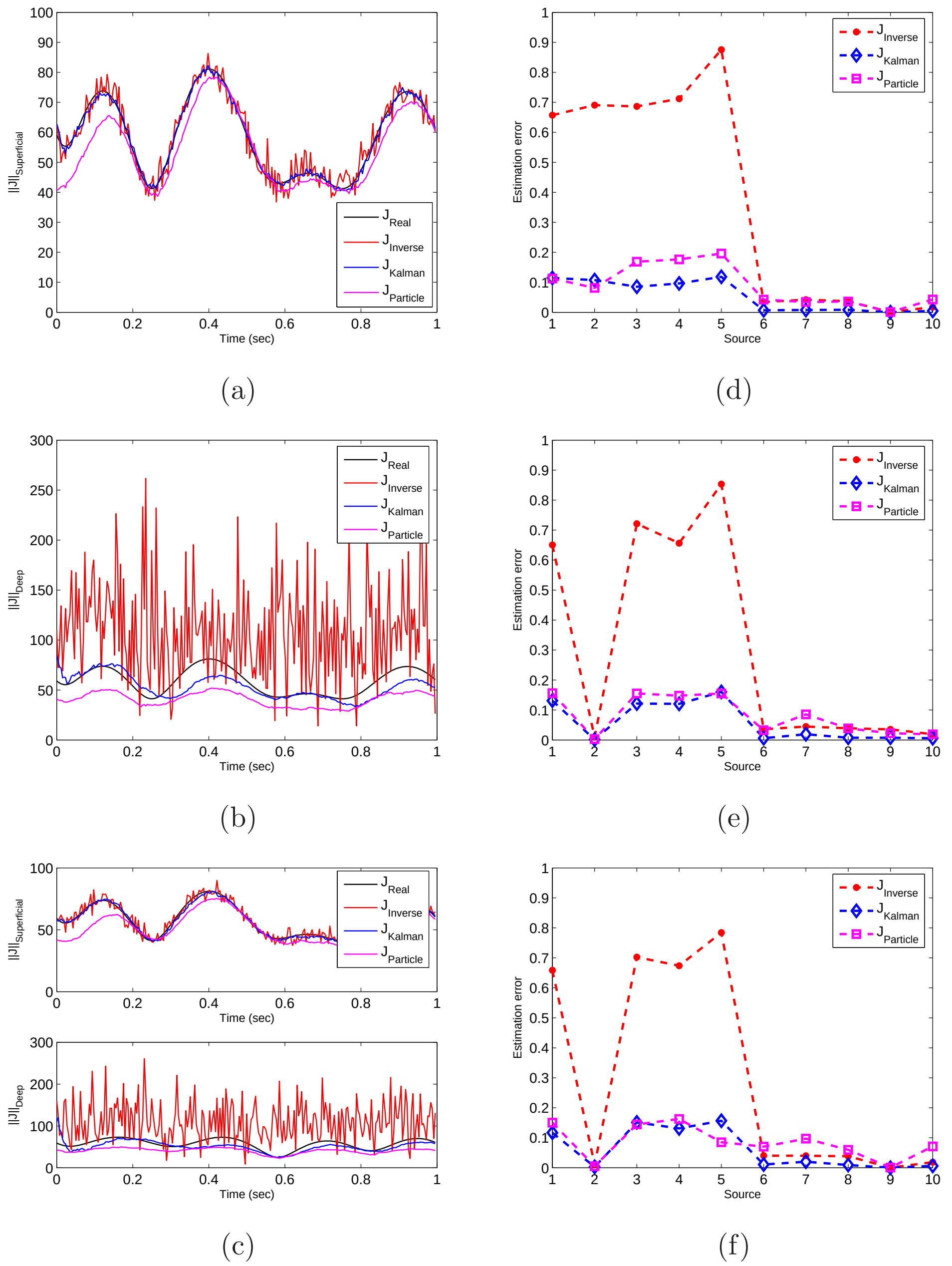

Figure 4.3: Left column: Magnitude of the real and estimated sources, (a) single active superficial source, (b) single active deep source and (c) two uncorrelated active sources. Right column: Estimation error for the whole source space given by the three methods, (b) single superficial active source, (e) single deep active source and (e) two uncorrelated active sources.

Chapter 5

Discussion and Conclusion

The solution to the EEG inverse problem looks for estimates of the neural sources within the brain that generate the electric potentials recorded with electrodes placed at the scalp. Two techniques have been developed to solve this problem, dipole methods and distributed methods. Despite their differences in the type of solution given by these method (dipolar methods estimate the position, orientation and magnitude of a priory known very small number of neural sources, whereas distributed methods estimate the orientation and magnitude of a large number of sources placed at fixed locations in the region of the brain), they have a common characteristic, they produce instantaneous estimations of the EEG neural generators where the inherent time dynamic evolution of the sources and voltages are not taken into account. They lack of the ability for the tracking the time evolution of the EEG generators.

This work describes a new methodology for the solution to the EEG inverse problem considering the dynamic characteristic of the neural sources. We use the Kalman filter and the Particle filter for the tracking the EEG sources. The former is the best dynamic solution for tracking when the model is linear and the variables have a Gaussian distribution. When the model is non-linear or the variables are non-Gaussian analytical solutions are not easy to obtain, so Particle filters can be used to solve tracking problems. These dynamic solutions are based on the recursive estimation of the neural sources space which is updated each time given new noisy measurements.

Although the Particle filter is appropriate for applications of non-linear systems, or for space state with non-Gaussian distributions, in this work we use the algorithm for tracking the neural sources in the case of linear model and with Gaussian variables and Gaussian noise. This first step in the usage of the sequential Monte carlo method is important to identify its correct parameters in the context of the EEG source localization problem for future applications where non-linear and non-Gaussian models will be considered.

Although the potentiality of the particle filter is ideal for applications of non-linear systems, or for space state with non-Gaussian distributions in this work we use the algorithm for tracking the neural sources in the case of linear model with Gaussian variables and Gaussian noise, which is the same scenery than the Kalman Filter. This is because this thesis is the first attempt to address this problem and the objective was to fully understand the potential of the technique in the context of the classical estimation tools. In the future work we will extend the use of the Particle filter in more complicated setups as the non-linear EEG inverse problem (dipolar approach) or non Gaussian noise.

The methods were evaluated using a three-shell spherical head model in simulations of three typical neurophysiological conditions of the EEG generators, superficial sources, deep sources and uncorrelated separated sources. Results show that for a single active superficial source the proposed dynamic methods have a remarkable performance in terms of the source localization error and estimation error. In contrast, the performance in terms of the reconstruction error is not very good, which is suggested to be due to the underestimation of non active sources. Nevertheless, this is not a problem since non active sources do not have physiological meaning and because the performance in terms of data fit error is very high.

For a single active deep source the performance in the estimation error decreases slightly in comparison with the estimation of superficial sources. This is because the dynamic solutions have as initial conditions the linear inverse solution which underestimates deep sources. Regarding the source localization error, the Kalman filter and the Particle filter show a very high performance. On the other hand, the fact that the reconstruction error is not very high can be attributed to an underestimation of non active sources instead a bad performance of the methods. This is acceptable since the performance in the data fit error is very good and because the estimation error is very remarkable.

Finally, for the case of two uncorrelated source, one superficial an one deep, the performance in the localization error and estimation error is remarkable and similar to the results obtained in the individual cases. The reconstruction of the whole sources space is very good and the scalp electric potentials produced by the estimated sources are very similar to the measured noisy potentials.

Together with the dynamics solutions we compute the instantaneous linear inverse solution. Thus, we can conclude that the general performance of the dynamic EEG inverse solutions is better than the performance obtained with the classical instantaneous inverse solution.

In summary, we have demonstrated through numerical simulations that the application of the Kalman filter and the Particle filter for the tracking of the EEG sources is very promising. Nonetheless, the Kalman filter approach requires more investigation in three aspects, first the selection of the initial source space estimation, second the selection of the transition matrix that model how the neural sources evolve in the time and third a study of the implications of increasing the number of the sources making the problem undetermined. Likewise, in the Particle filter approach much more investigation is still necessary in several aspects. First, the selection of the initial posterior distribution. Second, the dynamic model that describes the time evolution of the sources. Third, a deep study of choosing the solution from the posterior distribution. Fourth, the parameter required for the operation of the filter if the model becomes non-linear. Fifth, the computational capability to solve a very high dimensional problem in real time. Finally, the increase of the number of sources and the correct tracking of the solution when the problem becomes undetermined.

In the future work we will extend the application the Particle filter for the dynamic estimation of the EEG sources in fully non-linear setups, for example we will incorporate the estimation of the position of the sources which makes the EEG inverse problem non-linear.

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